Simultaneous Spatial and Spectral Selective Excitation

CRAIG H. MEYER, JOHN M. PAULY, ALBERT MACOVSKI, AND DWIGHT G. NISHIMURA

Information Systems Laboratory, Stanford University, Stanford, California 94305

Received January 17, 1989; revised December 29, 1989

Using a k-space interpretation of small-tip excitation, a single excitation pulse has been designed that is simultaneously selective in space and resonant frequency. An analytic expression for the response of this pulse has been derived. The pulse has been implemented on a 1.5-T imaging system. The pulse has been applied to a rapid gradient–echo imaging sequence that forms both water and fat images within a breath-holding interval. These rapid images are free of the chemical shift artifacts at organ boundaries that typically afflict conventional rapid images. The pulse can be applied to a variety of other sequences, such as multislice water/fat sequences and rapid k-space scanning sequences. © 1990 Academic Press, Inc.

INTRODUCTION

For a number of imaging applications one would like to selectively examine a particular spatial slice and a particular spectral component of the object at the same time. The most important example of this is two-dimensional water/fat imaging. Water/fat imaging may be desirable as an end in itself, for example, as a tool for examining atherosclerotic plaque. It may also be desirable to select for water or fat to avoid image artifacts, such as those encountered in rapid imaging sequences. Rapid gradient–echo imaging sequences based on steady-state free precession, such as FLASH (1), GRASS (2), FFE (3), FAST (4), and SSFP (5), suffer from artifacts at water/fat boundaries. Rapid k-space scanning sequences can suffer intolerable shifts or blurring of either water or fat (6–8).

Many techniques for forming water/fat images using spectrally selective excitation sequences have been studied. Most of these techniques combine a spatially selective pulse with an additional spectrally selective pulse (9, 10); multislice acquisition is impossible with these techniques. One recent technique uses two offset spatially selective pulses (11, 12). However, for many applications a single pulse that is simultaneously spectrally selective and spatially selective is preferable to a combination of pulses. Using the k-space interpretation of small-tip excitation introduced by Pauly et al. (13), we have designed such a pulse (14, 15). We will refer to this kind of pulse as a spatial–spectral pulse.

In this paper we first review excitation k space. Then we derive theoretical expressions for the magnetization excited in the presence of oscillating gradients. Next we discuss the design of a spatial–spectral pulse for water/fat imaging. We present computer simulations and experimental results that verify the theory. We apply the pulse
to a rapid gradient–echo sequence and obtain water/fat images of a normal volunteer at 1.5 T. Finally, we discuss some of the issues, advantages, disadvantages, and applications of simultaneous spatial and spectral selective excitation.

EXCITATION k SPACE

Pauly et al. (13) introduced a k-space interpretation of small-tip-angle selective excitation. They showed that multidimensional selective excitation in the presence of time-varying gradients can be analyzed using Fourier transform theory. Using that interpretation, they designed and implemented selective excitation pulses that were selective in two spatial dimensions. This k-space interpretation of excitation provides a number of the same conceptual advantages as the well-known k-space interpretation of the readout mode of magnetic resonance imaging (MRI) (16–20), although it differs in some important respects.

We first summarize the k-space interpretation of selective excitation by revisiting some results from (13). Using the well-known small-tip-angle approximation, the Bloch equation can be solved to give the following expression for the transverse magnetization:

$$M_{xy}(r) = i \gamma M_0(r) \int k W(k) S(k) e^{i r \cdot k} dk,$$

where

$$W(k(t)) = \frac{B_1(t)}{|k(t)|},$$

$$S(k) = \int_0^T \{ \delta(k(t) - k)|k(t)| \} dt.$$  

We will define the relevant k-space variables shortly. W(k) is a weighting function in multidimensional k space. S(k) is a sampling grid in k-space. The factor |k| normalizes S(k) so that it is a unit-strength line delta. A unit-strength line delta defined as a line delta that integrates to unity along a unit-length path. The transverse magnetization excited is proportional to the product of the initial magnetization at the inverse Fourier transform of the product of W(k) and S(k). In designing a selective excitation pulse having a given spatial distribution and its associated transform, or designing a k trajectory such that S(k) provides an adequate k-space sampling grid. Then one can use W(k) as the Fourier transform of the main lobe of the desired transverse magnetization within the limits of the small-tip approximation. Once S(k) and W(k) are chosen, it is straightforward to determine the corresponding gradient and RF waveforms.

The problem of designing a pulse that is spatially and spectrally selective is similar to the problem of designing a pulse that is selective in two spatial dimensions (13). We wish to design a pulse that is selective in both the slice-selection direction, z, and the spectral direction, \( \omega \). We thus define k-space axes corresponding to \( z \) and \( \omega \) as follows:

$$k_z(t) = -R$$

Note that the integration domain is contrast to readout k space. It extends up to the observation time.

The constant R in Eq. [4] is a c constant, R. To design a pulse in the gyromagnetic ratio. In that units of distance. In our problem, different. We choose the units of R. This means that the numerical value of R might vary between the cases but is fixed for one case. In the current magnetization. When one i convert k to units of natural units.

Because the k trajectory is on an axis, one must oscillate the slice structure in \( (k_z, k_\omega) \) space. Variables. Our discussion will center upon sinusoidal k trajectories.

$$G_z(t) = \sin \Omega(t)$$

This G_z(t) corresponds to the frequency of the wave. The projection of this k trajectory is shown in Figure 1.

In this section, we derive the RF pulses. The results of this derivation are spectral pulses. The technique...
SPATIAL AND SPECTRAL SELECTIVE EXCITATION

Fig. 1. Excitation k-space trajectory for a spatial-spectral pulse with a sinusoidal slice-selection gradient. The trajectory ends at the origin.

\[ k_z(t) = -R \int_0^T G_z(s) ds \quad k_x(t) = t - T. \]  \[4\]

Note that the integration defining \( k_z \) ranges over the time remaining in the interval, in contrast to readout \( k \) space, where the integration ranges from the start of the interval up to the observation time.

The constant \( R \) in Eq. [4] is a constant that is defined differently for different design problems. To design a pulse in two spatial dimensions, we would set \( R \) equal to \( \gamma \), the gyromagnetic ratio. In that case the units of \( k_z \) would be the reciprocal of the units of distance. In our problem, we have two \( k \) space axes whose natural units are different. We choose the units of \( R \) so that \( k_z \) has the same units as \( k_x \), e.g., seconds. This means that the numerical value that we choose for \( R \) is arbitrary; we are in effect choosing a relation between the natural units of the axes. We will discuss how one might choose the value of \( R \) later. In the design of a pulse, \( R \) is used to compute the normalization factor, \(|k(t)|\), in Eqs. [2] and [3], and it is used to calculate the resulting magnetization. When one is just interested in the spatial slice width, one can convert \( k_z \) back to natural units by setting \( R = \gamma \).

Because the \( k \) trajectory is constrained to move linearly with time along the \( k_x \) axis, one must oscillate the slice-selection gradient to generate an adequate sampling structure in \((k_z, k_x)\) space. Various forms of oscillation could be used. Most of our discussion will center upon sinusoidal oscillation of the form

\[ G_z(t) = G \cos \Omega(t - T), \quad 0 \leq t \leq T. \]  \[5\]

This \( G_z(t) \) corresponds to the following \( k \) trajectory:

\[ k_z = \frac{RG}{\Omega} \sin \Omega(t - T) = \frac{RG}{\Omega} \sin \Omega k_x, \quad -T \leq k_x \leq 0. \]  \[6\]

This \( k \) trajectory is shown in Fig. 1. Rather than starting at the origin as in readout \( k \) space, it ends at the origin, as a consequence of Eq. [4].

THEORY

In this section, we derive the theoretical magnetization excited by a spatial-spectral pulse. The results of this derivation will help us in designing and applying spatial-spectral pulses. The techniques used in the derivation may be useful to the reader.
interested in the effects of applying an RF pulse in the presence of an oscillating gradient. The reader who is interested primarily in using spatial–spectral pulses can proceed to the following section without loss of continuity.

Using the small-tip-angle approximation, we will now calculate the magnetization excited by a spatial–spectral pulse having a sinusoidal slice-selection gradient. This magnetization is proportional to the product of $M_0(\omega, z)$ and the two-dimensional (2D) inverse Fourier transform of the product of $S(k)$ and $W(k)$.

Let us define the one-dimensional (1D) Fourier transform and its inverse as

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-iu x} dx \quad \text{and} \quad f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(u) e^{iu x} du.$$  \[7\]

We designate 1D forward and inverse transforms by $\mathcal{F}\{\}$ and $\mathcal{F}^{-1}\{\}$, respectively, and their 2D counterparts by $\mathcal{F}^2\{\}$ and $\mathcal{F}^{-2}\{\}$. We use $\ast$ to represent 1D convolution and $\ast\ast$ to represent 2D convolution. We define the rectangle function such that $\text{rect}(x) = 1$ for $|x| < \frac{1}{2}$.

First we study the inverse Fourier transform of the unit-strength sinusoidal line delta, $S(k)$. We define the delta as

$$S(k) = VR^2G^2\cos^2\Omega k_w + 1 \delta\left(k_z - \frac{RG}{\Omega} \sin \Omega k_w\right).$$  \[8\]

Let $S(k)$ extend over the infinite interval $-\infty < k_w < \infty$. Any realizable $S(k)$ will of course be of finite extent along the $k_w$ axis; we will account for this in $W(k)$. $S(k)$ is periodic along the $k_w$ axis with period $2\pi/\Omega$. We can decompose it into a 1D Fourier series with $k_z$ as a parameter (21). We calculate the series coefficients, $C_n$, as

$$C_n(k_z) = \frac{\Omega}{2\pi} \int_{\pi/\Omega}^{\pi/\Omega} VR^2G^2\cos^2\Omega k_w + 1 \delta\left(k_z - \frac{RG}{\Omega} \sin \Omega k_w\right) e^{in\omega} \omega dk_w.$$  \[9\]

We want to express the argument of the delta function in the form $k_w - f(k_w)$, which involves inverting the sin function. Because $\sin^{-1}$ is a multivalued function, we must decompose this integral into four subintegrals, each ranging over a quarter of a cycle of $\sin \Omega k_w$. The first subintegral is

$$\frac{\Omega}{2\pi} \int_{\pi/\Omega}^{\pi/\Omega} S(k_z, k_w) e^{in\omega} \omega dk_w$$

$$= \int_{\pi/\Omega}^{\pi/\Omega} \beta(k_z) \delta\left(k_w - \frac{\pi}{\Omega} - \frac{1}{\Omega} \sin^{-1}\left(\frac{\Omega k_z}{RG}\right)\right) e^{in\omega} \omega dk_w$$  \[10\]

$$= \beta(k_z) e^{-in\pi} e^{in\sin^{-1}(1/2RG)k_z},$$  \[11\]

where

$$\beta(k_z) = \frac{\Omega}{2\pi} \sqrt{1 - \left(\frac{\Omega k_z}{RG}\right)^2} + \frac{1}{R^2G^2} = \frac{\Omega}{2\pi} \sqrt{1 - \left(\frac{\Omega k_z}{ARG}\right)^2}$$  \[12\]

In Eq. [10] we have replaced delta function that is equivalent subintegrals the form of the original argument of the delta varies, a Table I summarizes the results particular range of $k_z$. We add ing Fourier series coefficients:

$$C_n(k_z) = \begin{cases} \beta(k_z)2 \text{ re} \\ \beta(k_z)2i \text{ re} \end{cases}$$

Substituting Eq. [12] into Eq. [1]

$$\mathcal{F}^{-1}\{C\}$$

where $K_1$ is a constant and

The Fourier series for $S(k)$ is

$$S(k_z)$$
in the presence of an oscillating using spatial–spectral pulses can tinuity.

now calculate the magnetization idal slice-selection gradient. This $g_0(\omega, z)$ and the two-dimensional \( k \) and \( W(k) \).

transform and its inverse as

$$
\frac{1}{2\pi} \int_{-\pi}^{\pi} F(u)e^{iux} du. [7]
$$

by \( \mathcal{F} \{ \cdot \} \) and \( \mathcal{F}^{-1} \{ \cdot \} \), respectively. We use \( * \) to represent 1D

We define the rectangle function

the unit-strength sinusoidal line

$$
\frac{RG}{\Omega} \sin \Omega k_u.
$$

< \infty. Any realizable \( S(k) \) will otrect for this in \( W(k) \). \( S(k) \) is decompose it into a 1D Fourier series coefficients, \( C_n \), as

$$
-\frac{RG}{\Omega} \sin \Omega k_u e^{-itwz} dw. [9]
$$

unction in the form \( k_u - f(k_z) \), \( \sin^{-1} \) is a multivalued function, als, each ranging over a quarter

$$
n^{-1} \left( \frac{\Omega}{RG} k_z \right) e^{intw} dw. [10]
$$

$$
\frac{2}{\pi} \sqrt{1 - \left( \frac{\Omega k_z}{2RG} \right)^2}
$$

$$
\frac{2}{\pi} \sqrt{1 - \left( \frac{\Omega k_z}{2RG} \right)^2}
$$

<table>
<thead>
<tr>
<th>Limits</th>
<th>Argument of ( \delta(\cdot) )</th>
<th>Result</th>
<th>Range of ( k_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\frac{\pi}{2\Omega} \leq k_u \leq \frac{\pi}{2\Omega})</td>
<td>(k_u - \frac{\Omega}{2\Omega} \sin^{-1} \left( \frac{\Omega}{RG} k_z \right))</td>
<td>(\beta(k_z) e^{-\text{i}n\pi k_w} e^{-\text{i}n\sin^{-1} \left( \frac{\Omega}{RG} k_z \right)})</td>
<td>(-\frac{\Omega}{2} \leq k_z \leq 0)</td>
</tr>
<tr>
<td>(0 \leq k_z \leq \frac{\pi\Omega}{2})</td>
<td>(k_u - \frac{\Omega}{2\Omega} \sin^{-1} \left( \frac{\Omega}{RG} k_z \right))</td>
<td>(\beta(k_z) e^{\text{i}n\pi k_w} e^{\text{i}n\sin^{-1} \left( \frac{\Omega}{RG} k_z \right)})</td>
<td>(-\frac{\Omega}{2} \leq k_z \leq 0)</td>
</tr>
<tr>
<td>(\frac{\pi}{2\Omega} \leq k_u \leq \frac{\pi}{2\Omega})</td>
<td>(k_u - \frac{\Omega}{2\Omega} \sin^{-1} \left( \frac{\Omega}{RG} k_z \right))</td>
<td>(\beta(k_z) e^{-\text{i}n\pi k_w} e^{-\text{i}n\sin^{-1} \left( \frac{\Omega}{RG} k_z \right)})</td>
<td>(0 \leq k_z \leq \frac{\Omega}{2})</td>
</tr>
<tr>
<td>(0 \leq k_u \leq \frac{\pi}{2\Omega})</td>
<td>(k_u - \frac{\Omega}{2\Omega} \sin^{-1} \left( \frac{\Omega}{RG} k_z \right))</td>
<td>(\beta(k_z) e^{\text{i}n\pi k_w} e^{\text{i}n\sin^{-1} \left( \frac{\Omega}{RG} k_z \right)})</td>
<td>(0 \leq k_z \leq \frac{\Omega}{2})</td>
</tr>
</tbody>
</table>

and

$$
A = \sqrt{1 + \frac{1}{R^2G^2}}. [13]
$$

In Eq. [10] we have replaced the unit delta function of Eqs. [8] and [9] with a unit delta function that is equivalent within the limits of integration. For the remaining subintegrals the form of the equivalent unit delta function is slightly different; the argument of the delta varies, although the normalization factor remains the same.

Table 1 summarizes the results for the four subintegrals. Each integral is valid for a particular range of \( k_z \). We add the integrals in each \( k_z \) range to determine the following Fourier series coefficients:

$$
C_n(k_z) = \begin{cases} 
\beta(k_z) 2 \text{ rect} \left( \frac{\Omega k_z}{2RG} \right) \cos \left[ \frac{n \sin^{-1} \left( \frac{\Omega k_z}{RG} \right)}{\Omega} \right] & \text{for } n \text{ even} \\
\beta(k_z) 2i \text{ rect} \left( \frac{\Omega k_z}{2RG} \right) \sin \left[ \frac{n \sin^{-1} \left( \frac{\Omega k_z}{RG} \right)}{\Omega} \right] & \text{for } n \text{ odd}
\end{cases} \quad [14]
$$

Substituting Eq. [12] into Eq. [14] and performing an inverse Fourier transform yield

$$
\mathcal{F}^{-1} \{ C_n(k_z) \} = K_1 \frac{J_1(Az')}{2A\pi} * J_n(\pi'), \quad [15]
$$

where \( K_1 \) is a constant and

$$
z' = \frac{RGz}{\Omega}. \quad [16]
$$

The Fourier series for \( S(k) \) is

$$
S(k_z, k_w) = \sum_{n=-\infty}^{\infty} C_n(k_z) e^{-inw\Omega}. \quad [17]
$$
The inverse Fourier transform of this series representation of $S(k)$ is

$$
2\mathcal{F}^{-1}\{S(k_x, k_z)\} = \sum_{n=-\infty}^{\infty} \mathcal{F}^{-1}\{C_n(k_z)\} \delta(\omega - n\Omega) = K_1 \frac{J_1(\Delta z')}{2\Delta z'} \sum_{n=-\infty}^{\infty} J_n(z') \delta(\omega - n\Omega). \tag{18}
$$

The transform of $S(k)$ is thus a series of weighted line deltas parallel to the $k_z$ axis and separated by $\Omega$, the gradient modulation frequency.

Assuming $W(k_x, k_z) = W(k_z)W(k_x)$, the resulting transverse magnetization excited by the pulse is

$$
M_{xy}(z, \omega) = i2\pi \gamma M_0(z, \omega) \mathcal{F}^{-1}\{W(k_x)\}
\times \delta(k_z) \sum_{n=-\infty}^{\infty} \mathcal{F}^{-1}\{C_n(k_z)W(k_z)\} \delta(\omega - n\Omega). \tag{19}
$$


$$
M_{xy}(z, \omega) = K_3 M_0(z, \omega) \mathcal{F}^{-1}\{W(k_z)\} \times \delta(k_z) \sum_{n=-\infty}^{\infty} \left[ \mathcal{F}^{-1}\{W(k_z)\} \ast \frac{J_1(\Delta z')}{2\Delta z'} \ast J_n(z') \right] \delta(\omega - n\Omega), \tag{20}
$$

where $K_3$ is a constant. This result shows that the $n$th sidelobe has the form of a smoothed $J_n$ in the $z$ direction and the form of the inverse transform of $W(k_z)$ in the $\omega$ direction. Assuming that the spacing of the spectral islands, $\Omega$, is large compared to the width of $\mathcal{F}^{-1}\{W(k_z)\}$, Eq. [20] permits straightforward calculation of $M_{xy}$ for separable $W$.

One can also calculate $M_{xy}$ by looking at the solution of the Bloch equation in the time domain. This solution can be written as $(13, 22)$

$$
M_{xy}(z, \omega) = i\gamma M_0(z, \omega) \int_0^T B_1(t)e^{i(\gamma G_z t + \sin\theta t)T} e^{i\omega(t-T)} dt. \tag{21}
$$

Using the identity

$$
e^{i\sin\theta t} = \sum_{n=-\infty}^{\infty} J_n(\alpha) e^{in\Omega t}, \tag{22}
$$

we can rewrite Eq. [21] as

$$
M_{xy}(z, \omega) = i\gamma M_0(z, \omega) \sum_{n=-\infty}^{\infty} J_n(\frac{\gamma G_z}{\Omega}) \int_0^T B_1(t)e^{i(n\Omega + \omega)(t-T)} dt. \tag{23}
$$

Here we also arrive at an infinite sum containing $J_n$. This equation can be used to calculate $M_{xy}$, although Eq. [20] is more practical for most purposes. To approximately calculate the sidelobe at $\omega = k\Omega$, the infinite sum in Eq. [23] can be replaced by a finite sum centered at $n = -k$. The number of terms required depends upon how rapidly $B_1(t)$ varies.

To this point we have assumed a spatial model. This is a convenient way to express and simulate behavior we can also calculate the magnetization employed. The derivation is similar to the result here.

Assume that $G_z(t)$ is a square magnetization can then be expressed

$$
M_{xy}(z, \omega) = K_3 M_0(z, \omega) \mathcal{F}^{-1}\{\}
\times \delta(\omega - n\Omega),
$$

where $K_3$ is a constant and

$$
a_n(z) = \begin{cases} \text{sinc} & \\ \text{sinc} \end{cases} \tag{24}
$$

In this equation $\text{sinc}(x) = \sin(x)/x$.

We now have expressions for the spatial slice-selection design of pulses that are similar to the slice-selection gradient, $G_z$, of the desired and undesired coil amplitude and frequency of the RF envelope, which deter the spectral slice profiles. The desired pulses to shift them in space and times in the context of the design.

We wish to design a slice-selected against fat protons, or via a difference frequency between the field inhomogeneity across the.

The main requirement for the is in some manner so that the $k$, arguments, makes intuitive $G_z$. We know that any long $T_2$ gradients. We want to add a $G_z$ to prevent chemical shift from this to slice the gradient. of slew-rate limitations.
To this point we have assumed that the slice-selection gradient varies in a sinusoidal fashion. This is a convenient waveform to generate experimentally, and all of our experiments and simulations have been performed using such a gradient. However, we can also calculate the magnetization resulting when a square-wave gradient is employed. The derivation is similar to that in the sinusoidal case, so we just present the result here.

Assume that \( G_z(t) \) is a square wave of amplitude \( G \) and period \( T \). The resulting magnetization can then be expressed by

\[
M_{xy}(z, \omega) = K_3 M_0(z, \omega) \mathcal{F}^{-1} \{ W(k_z) \} 
\times \delta(k_z) \delta(\omega - n \Omega),
\]

where \( K_3 \) is a constant and

\[
a_n(z) = \begin{cases} 
\frac{\sin\left(\frac{z'' + \frac{n}{2}}{2}\right) + \sin\left(\frac{z'' - \frac{n}{2}}{2}\right)}{2} & \text{for } n \text{ even} \\
\frac{\sin\left(\frac{z'' + \frac{n}{2}}{2}\right) - \sin\left(\frac{z'' - \frac{n}{2}}{2}\right)}{2} & \text{for } n \text{ odd}.
\end{cases}
\]

In this equation \( \sin\left(\frac{x}{2}\right) = (\sin \pi x)/(\pi x) \) and \( z'' = (RGT/4\pi)z \).

**DESIGN CONSIDERATIONS**

We now have expressions for the magnetization excited by a pulse in the presence of an oscillating slice-selection gradient. Using these expressions, we can discuss the design of pulses that are simultaneously selective in space and resonant frequency. Some of the considerations in the design of these pulses are (1) the functional form of the slice-selection gradient, which determines the \( k \) trajectory, (2) the placement of the desired and undesired components relative to the frequency sidelobes, (3) the amplitude and frequency of the slice-selection gradient, (4) the functional form of the RF envelope, which determines the \( k \)-space weighting and thus the spatial and spectral slice profiles, (5) the length of the pulse, and (6) the modulation of these pulses to shift them in space and frequency. In this section we discuss these considerations in the context of the design of a water/fat-selective slice-selection pulse.

We wish to design a slice-selective pulse that excites water protons and discriminates against fat protons, or vice versa. The target field strength is 1.5 T, where the difference frequency between water and fat is about 230 Hz. We assume that the main field inhomogeneity across the slice is less than ±1 ppm.

The main requirement for the slice-selection gradient, \( G_z \), is simply that it oscillate in some manner so that the \( k_z \) axis is sampled. Even without referring to \( k \)-space arguments, it makes intuitive sense that a spatial–spectral pulse would need such a \( G_z \). We know that any long RF pulse will be spectrally selective in the absence of gradients. We want to add a \( G_z \) while preserving this spectral selectivity. One way to prevent chemical shift from simply mapping into a shift in the slice-selection direction is to oscillate the gradient. True square-wave gradients are impractical because of slew-rate limitations. Here we choose sinusoidal gradients, which are simple to
generate and allow us to verify the theory of the previous section. Trapezoidal gradients are another practical choice.

Before choosing a gradient modulation frequency, we must decide where in the spectrum to place the undesired component, relative to the frequency sidelobes of the excitation pulse. For this discussion, let us assume water is the desired component and fat the undesired component. Water is placed at the central lobe. The simplest method, and the one that we adopt here, is to place fat at the null between the main lobe and the first sidelobe. The required gradient modulation frequency for this method is twice the water/fat difference frequency (460 Hz for 1.5 T). This method leads to the maximum frequency separation between the sidelobes, which has two positive effects: (1) the transition band between water and fat is broad, so that the pulse can be short and (2) the water/fat separation is relatively insensitive to main field inhomogeneity. The main disadvantage of this method is that the minimum slice width is limited, both because the maximum gradient amplitude is limited by slew-rate constraints and because the short gradient period limits the extent of the k_z excursion. If the slice width or slice profile is not acceptable using the above method, then fat can be placed elsewhere (e.g., between the first and second sidelobe) or greater gradient power can be used. It may be advantageous in some circumstances to place fat closer to an odd sidelobe than to an even sidelobe, because the odd symmetry of odd sidelobes results in a decreased integral across the slice.

The placement of fat is strongly influenced by the gradient power and the field strength of the system. When gradient power is sufficiently high, placing fat at the closest null will generally be preferable. For a fixed gradient power, the achievable slice width decreases as the field strength increases. At 1.5 T and using the gradient power available on commercial whole-body imagers, it is possible to achieve slice widths on the order of 1.0 to 1.5 cm placing fat at the closest null. This slice width is adequate for many applications. At higher field strengths it may be necessary to place fat elsewhere or to use stronger gradients. At field strengths below 1.0 T placing fat at the closest null should suffice for most applications.

Now that we have chosen the slice-selection gradient as a 460-Hz cosine, we can study the RF envelope. The spatial and spectral weighting can be chosen independently. Here we present a simple pulse with Gaussian weighting on both the k_z and k_x axes. This weighting leads to compact spectral and spatial slice profiles, which are Gaussian in shape in the small-tip-angle regime. In determining the equations for the pulse, we must choose the value of R, the arbitrary scaling factor for the k_z axis. We want the spatial profile of the main lobe to be as close to the Fourier transform of W(k_z) as possible. It is clear from Eq. (14) with n = 0 that we want A = 1, because C_n(k_z) would then just be a rectangle function the width of W(k_z). We choose RG to be arbitrarily large relative to one, leading to the expression

\[
|k(t)| = \sqrt{k_z^2 + k_x^2} = \sqrt{(RG)^2 + 1} \\
= \sqrt{[RG \cos(\Omega(t - T))]^2 + 1} \\
= RG \cos(\Omega(t - T)), \quad \text{as} \quad 1/RG \rightarrow 0.
\]

Figure 2 shows the RF and gradient.

To achieve more-rectangular axes. For flip angles in the nor weighting can also be determination of slice profiles (23–27).

With the form of the excitatic U, V, and G remains. T must be tion band is narrower than the bands lead to greater immunity that the RF amplifier can rema phasing and T_z decay during the isochromat with a frequency of the pulse, assuming that the weig T_z decay can be thought of as weighting. We typically choose 13.0 ms. U and V are chosen s without excessive ringing. G is a maximum gradient strength ach a 460-Hz sinusoid, which allows

Now the specifications are cons fat, assuming that the transm produce a modulated version simply multiplies B_1(t) from Eq for a pulse that has been modula

Modulating the pulse to produc. To offset the slice by \Delta \tau \epsilon by approximately 1.5 slice width
Vious section. Trapezoidal gradient, we must decide where in the ve to the frequency sidelobes of the water is the desired component at the central lobe. The simplest fat at the null between the main modulation frequency for this 460 Hz for 1.5 T). This method en the sidelobes, which has two iter and fat is broad, so that the is relatively insensitive to main is method is that the minimum gradient amplitude is limited by period limits the extent of the k2, eptable using the above method, e first and second sidelobe) or ntagious in some circumstances lead to the odd sym- across the slice.
he gradient power and the field efficiently high, placing fat at the l gradient power, the achievable At 1.5 T and using the gradient rs, it is possible to achieve slice e closest null. This slice width is gths it may be necessary to place engths below 1.0 T placing fat at
ient as a 460-Hz cosine, we can weighting on both the k2 and d spatial slice profiles, which are determining the equations for the scaling factor for the k2 axis. We lose to the Fourier transform of 0 that we want A = 1, because width of W(k2). We choose RG expression

\[ 1/\text{RG} \rightarrow 0. \]  

[26]  

[27]  

\[ \text{SPATIAL AND SPECTRAL SELECTIVE EXCITATION} \]  

\[ B_1(t) = B_1 e^{-\frac{\sqrt{2}(\sin(\gamma t - T)/U)^2}{1}} e^{-\frac{\sqrt{2}(1-(T/2))/V^2}{1}} \cos \Omega(t - T) \]  

[28]  

\[ G_z(t) = G \cos \Omega(t - T), \quad 0 \leq t \leq T. \]  

[29]  

Fig. 2. Rapid gradient-echo pulse sequence using a spatial-spectral pulse with Gaussian k-space weighting along both k2 and kz.

The resulting equations for the pulse are

Figure 2 shows the RF and gradient waveforms for this pulse.

To achieve more-rectangular pulse profiles, one can use sinc weighting on either axis. For flip angles in the nonlinear regime, the form of the spectral and spatial weighting can also be determined by a 2D extension of 1D techniques for optimization of slice profiles (23–27).

With the form of the excitation determined, only the choice of the parameters T, U, V, and G remains. T must be chosen to be long enough so that the spectral transition band is narrower than the water/fat difference frequency. Narrower transition bands lead to greater immunity to main field inhomogeneity. The maximum time that the RF amplifier can remain unblanked sets an upper limit on T. Spectral dephasing and T2 decay during the pulse also limit T. In the small-tip-angle regime, an isochromat with a frequency offset of \( \Delta \omega \) acquires a phase factor of \( e^{-i(\Delta \omega T)^2} \) during the pulse, assuming that the weighting is symmetrical about the midpoint of the pulse. T2 decay can be thought of as adding the weighting factor \( e^{-i(T-i)/T_2} \) to the desired weighting. We typically choose T to be equal to six cycles of \( G_z \), which is equal to 13.0 ms. U and V are chosen such that the slice profiles are as narrow as possible without excessive ringing. G is simply chosen to achieve the desired slice width. The maximum gradient strength achievable on our experimental system is 0.7 G/cm for a 460-Hz sinusoid, which allows a minimum slice width of about 1.2 cm.

Now the specifications are complete for a pulse that excites water without exciting fat, assuming that the transmitter is tuned to the water frequency. It is a simple matter to produce a modulated version of this pulse that excites fat instead of water; one simply multiplies \( B_1(t) \) from Eq. [28] by \( e^{i\omega t} \). The I- and Q-channel RF waveforms for a pulse that has been modulated by 230 Hz are shown in Fig. 3a.

Modulating the pulse to produce a spatially offset slice is only slightly more complicated. To offset the slice by \( \Delta z \) one multiplies \( B_1(t) \) by \( e^{i\omega z(t)/\Delta z} \). A pulse that is offset by approximately 1.5 slice widths is shown in Fig. 3b.
SIMULATION RESULTS

In the previous section we discussed a spatially selective pulse designed to selectively excite either water or fat at 1.5 T. In this section we study the behavior of this pulse using a numerical simulation of the Bloch equation. We then compare the simulation results with the results predicted by the small-tip-angle theory. The simulations are performed on the pulse shown in Fig. 2. The object is assumed to be infinite and uniform, and relaxation is neglected.

Figure 4 shows the simulated $|M_{\omega z}|$ following a 90° pulse as a function of $z$ and $\omega$. The center of the figure corresponds to $z = 0$ and $\omega = 0$. The form of the response agrees well with the small-tip-angle theory. Along the $\omega$ axis the islands are spaced at the gradient modulation frequency, $\Omega$. For the central lobe both the spectral and spatial slice profiles are Gaussian way between the central lobe and broad null. Figure 5 compares $|M_{\omega z}|$ versus $z$ at resonant frequency of the spatial–spectral pulse of Fig. 2, 45.5 dB.

In computing the theoretical $K_2\mathcal{F}^{-1}$ contribution from other sidelobe discretely:

Figure 6 compares the main-lobe for 30° and 90° flip angles. The tip pulse was applied along the $x$ axis:

**Fig. 4.** $|M_{\omega z}|$ versus $z$ and $\omega$ from a computer simulation of the spatial–spectral pulse of Fig. 2 at a flip angle of 90°.

**Fig. 5.** $|M_{\omega z}|$ versus $z$ at resonant frequency of the spatial–spectral pulse of Fig. 2, 45.5 dB.

**Fig. 6.** $M_z$ versus $z$ at two different flip the theoretical magnetization calculated simulation of the pulse. (a) $M_z$ versus $z$ at tip approximation is quite accurate at 30°, but the
modulating the RF waveform of Fig. 2 to
when the transmitter is tuned to the water
by modulating the RF waveform of Fig. 2

selective pulse designed to selec-
tion we study the behavior of this
equation. We then compare the
small-tip-angle theory. The simu-
2. The object is assumed to be
90° pulse as a function of z and ω.
ω = 0. The form of the response
the ω axis the islands are spaced at
central lobe both the spectral and

spatial slice profiles are Gaussian. The undesired component should be placed half-
way between the central lobe and the first sidelobe along the ω axis, where there is a
broad null. Figure 5 compares \(|M_{ox}|\) versus z for the desired and undesired com-
ponents. The relative suppression for the magnitude of the integral across the slice is
45.5 dB.

In computing the theoretical spatial response at the rth sidelobe, we ignored the
contribution from other sidelobes in Eq. [20]. We evaluated the following convolu-
tion discretely:

\[
K_2 \mathcal{F}^{-1} \left\{ W(k) \right\} \ast \frac{J_1(Az')} {2Az'} \ast J_n(z').
\]

Figure 6 compares the main-lobe theoretical \(M_z\) versus z with the simulated response
for 30° and 90° flip angles. The only significant magnetization was in \(M_y\), because the
pulse was applied along the x axis. The simulated data were corrected for an arbitrary

Fig. 5. \(|M_{ox}|\) versus z at resonant frequencies corresponding to water and fat from a computer simu-
lation of the spatial–spectral pulse of Fig. 2 at a flip angle of 90°. The suppression of fat relative to water is
45.5 dB.

Fig. 6. \(M_z\) versus z at two different flip angles for the spatial–spectral pulse of Fig. 2. The solid lines are
the theoretical magnetization calculated by discrete convolution. The open circles are from a numerical
simulation of the pulse. (a) \(M_z\) versus z at a flip angle of 30°. The theoretical expression based on the small-
tip approximation is quite accurate at 30°. (b) \(M_z\) versus z at a flip angle of 90°. The small-tip approxima-
tion begins to break down at 90°, but the slice profile is still quite usable.
constant phase factor, but not for any spatially varying phase. The theoretical $M_y$ is almost indistinguishable from the simulated $M_y$ at 30°, which indicates that small-tip-angle approximation is accurate. The response at 90° illustrates that the small-tip-angle approximation is beginning to break down, but the resulting slice profile is actually more rectangular than the 30° profile. This is consistent with current design practice for one-dimensional slice-selection pulses, where useful 90° pulses are often designed by Fourier transform methods, even though the small-tip-angle approximation is not strictly valid at 90°.

Figure 7 compares the theoretical $M_y$ versus $z$ with the 30° simulated response at the main lobe and the first three sidelobes. Once again the agreement is quite good. The even sidelobes have even symmetry and the odd sidelobes have odd symmetry, as predicted.

EXPERIMENTAL RESULTS

The pulse was implemented on a General Electric Signa 1.5-T whole-body imaging system with self-shielded gradient coils. To experimentally verify the sidelobe behav-
Fig. 2. The magnetization is shown at the \( \omega \) axis. The solid lines are the theoretical values and are from a numerical simulation of the 1) Third sidelobe.

...
Fig. 9. Experimental axial head images acquired using the rapid gradient-echo sequence of Fig. 2. The water-selective RF pulse of Fig. 2 was alternated with the fat-selective RF pulse of Fig. 3a so that the effective repetition time of either water or fat was twice the actual repetition time of the transmitter. The total acquisition time for both images was 16 s. (A) Water image: the optic nerves are clearly visible because the orbital fat is suppressed. (B) Fat image.

does not perturb the water protons. By analogy to multislice acquisition mode, we call this multispectrum acquisition mode. With one average, both water and fat images were formed in 16 s. The slice width was approximately 1.2 cm, using a 0.7-G/cm, 460-Hz slice-selection gradient. Figure 9 shows axial images of the head of a normal volunteer obtained with this sequence. The optic nerves are clearly visible in the water image, because the orbital fat is suppressed. The orbital and subcutaneous fat are visible in the fat image. Figure 10 shows axial images of the body of a normal volunteer. Note the absence of images that are not selective for defined and there are no visible gradient is well-behaved in th compensation.

We designed a single pulse tl
We designed this pulse using tl
rapid gradient-echo sequence of Fig. 2. The t-selective RF pulse of Fig. 3a so that the usual repetition time of the transmitter. The optic nerves are clearly visible because

To multislice acquisition mode, we average, both water and fat approximately 1.2 cm, using a 0.7-G/axial images of the head of a optic nerves are clearly visible in red. The orbital and subcutaneous images of the body of a normal volunteer. Note the absence of artifacts at the organ boundaries, in contrast to rapid images that are not selective for water or fat. Note also that the blood vessels are well defined and there are no visible flow artifacts. This indicates that the slice-selection gradient is well-behaved in the presence of flow, without any lobes added for flow compensation.

**DISCUSSION**

We designed a single pulse that is simultaneously spatially and spectrally selective. We designed this pulse using the k-space interpretation of small-tip-angle excitation,
an analysis technique developed by Pauly and his co-workers. We derived the inverse Fourier transform of infinite sinusoidal and square-wave line deltas to develop theoretical expressions for the magnetization excited in the presence of an oscillating slice-selection gradient. One useful example of a spatial–spectral pulse is a spatially selective water/fat pulse. We discussed the design of such a pulse for a whole-body 1.5-T imaging system. We presented computer simulations and experimental results and verified that the theoretical expressions are valid. We then applied the pulse to a rapid gradient–echo imaging sequence. The resulting water/fat images are free of the chemical shift artifacts commonly associated with such rapid imaging sequences.

There are a number of advantages of the spatial–spectral pulse. We have shown experimentally that it is useful in rapid gradient–echo imaging sequences, particularly in the abdomen. Problems with interference between water and fat at organ boundaries disappear. Because only one spectral element is excited at a time, multispectrum imaging is possible, as demonstrated by the alternation of the water and fat excitations in the gradient–echo sequence. The moments of the slice-selection gradient are small, so flow artifacts are minimized. At the end of pulse the magnetization is inherently refocused and the short refocusing interval is only necessary because of the finite gradient switching time. There is no misregistration in the z direction between the water and fat slices. Unlike water/fat sequences with one spatially selective pulse and a separate spectrally selective pulse mode. This pulse is unique in it the same set of spatial slices; the rely on the spatial misregistration multislice and multispectrum spectral component in sequence.

There are of course some prat suppression that can be achieved a system is the primary limit. So the pulse somewhat more difficult currents, a phase shift of the slice. The length of the pulse is limited that the RF amplifier can remain may be impractical for extr

This pulse is especially useful planar imaging or square-spiral make water/fat excitation a need cardiac movies, where the several times during a heartbeat. The cardiac movies, because it suppr dephase the flowing blood. We h: imaging, both to form water-sel
o-workers. We derived the inverse \(\tau\)-wave line deltas to develop the idea of an oscillating slice-spectral pulse is a spatially selected pulse for a whole-body 1.5-T system and experimental results and We then applied the pulse to a set of images are free of the such rapid imaging sequences.

A-spectral pulse. We have shown 10 imaging sequences, particularly with water and fat at organ boundaries excited at a time, multispectrum ion of the water and fat excitations where the slice-selection gradient is small, the magnetization is inherently necessary because of the finite in the \(z\) direction between the one spatially selective pulse and a separate spectrally selective pulse, this pulse can be used in a multislice acquisition mode. This pulse is unique in its ability to perform multislice water/fat imaging of the same set of spatial slices; the other published multislice water/fat pulse sequences rely on the spatial misregistration of water and fat. One can even perform a combined multislice and multispectrum experiment, where one excites each desired spatial and spectral component in sequence.

There are of course some practical limits to the slice width, slice profile, and spectral suppression that can be achieved with this pulse. The gradient slew rate available on a system is the primary limit. Severe eddy currents might make implementation of the pulse somewhat more difficult on some systems, but the main effect of such eddy currents, a phase shift of the slice-selection gradient, can easily be compensated for. The length of the pulse is limited by \(T_2\) decay and by the maximum length of time that the RF amplifier can remain unblanked. Finally, any water/fat-selective excitation may be impractical for extremely low-field systems, because \(\Delta\omega\) is too small.

This pulse is especially useful in rapid \(k\)-space scanning sequences such as echo planar imaging or square-spiral imaging. In such sequences the long readout times make water/fat excitation a necessity. One common use of these sequences is to form cardiac movies, where the sequence is repeated, with small-tip-angle excitations, several times during a heartbeat. The spatial–spectral pulse is particularly useful in these cardiac movies, because it suppresses fat without a 180° pulse and because it does not dephase the flowing blood. We have applied the spatial–spectral pulse to square-spiral imaging, both to form water-selective cardiac movies and to form water/fat multi-

---

**Fig. 10—Continued**
slice fast images of the heart and the abdomen (15). We will report further on this work in a later paper.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the support of the GE Medical Systems Group under Grant 22-84 and the National Institutes of Health under Grant 1RO1 HL-39297.

REFERENCES


Improved Quantification of Signal Acquisition of Prior Know

A. A. De Gi

Department of Appl
P.O. Box 30

Quantification of localized in excessive spectral overlap. In ac of the NMR pulse sequence on these problems, it comprises ut mental lineshapes: determination linewidths using model solution edge into a nonlinear least-squa greatly improved accuracy, pre spectra of rat brain, and enabl

© 1990 Academic Press, Inc.

To study the pathology of hep and phosphorus in vivo NMR sp determination of small concen: spectral quantification is essenti minimization of peak heights or of peing excited peaks are not able t fitting the spectrum or the time c domain fitting by the LPSVD me (4), but for 3H MRS some prol lineshapes, the residual water si water suppression pulse sequenc ress is made in solving these pro spectral region of interest. This h

(7). The following special prob
e-1. Fitting in the frequency do
e-2. The signal-to-noise ratio (S
e-3. The experimental lineshape
ations (Lorentzian and/or Gauss mogeneity, motions of the anim;
e-4. In the case of strong overli ances of the fitting results will be
tween the fitted parameters.